Group Theoretic Geometry

Kazuhisa MAEHARA*

1 Introduction:

In this note we shall propose a construction of a new geometry called group theoretic geometry. Possibly it might be a variation of a representation theory of groups. We grasp the concept of a variety in this sense as follows: Given a field F, we patch subrings (R_i) of the field F with the same total quotient field $Q(R_i) = F$ in such a way that there exists a topological space $X = \cup_i U_i$ such that $U_i = \mathrm{Spec} R_i$. Here the subrings (R_i) form an inductive system and a generic point is represented by $F = \varinjlim R_i$. For an affine variety R_i and an irreducible divisor $V(f)$, the generic point of $V(f)$ is defined by the total quotient of $R_i/(f)$.

Let Γ be a profinite group and $\{H_i\}$ a family of profinite normal closed subgroups of Γ. We patch quotient groups Γ/H_i such that there exists a topological space $X = \cup_i U_i$, where $\pi_1(U_i) = \Gamma/H_i$. The Γ/H_i form a projective system and the limit is $\Gamma = \varprojlim \Gamma/H_i$. For an affine variety $U_i = \mathrm{Spec} R_i$ and a divisor D of U_i, $\pi_1(U_i)$ is a factor group of a profinite group Γ_D of a generic poit of D. We have $\Gamma_D = \varprojlim_{g \neq f} \pi_1(R_i[f^{-1}]/(g))$, where $V(g) = D$.

2 Affine scheme

Lemma 2.1. *Let A_p be a local ring and p a prime ideal. Let $\hat{A}_p$ be a completion of A_p. Then $\hat{A}_p \to A_p$ is faithfully flat and etale. Hence the homomorphism $\pi_1(\mathrm{Spec}\ \hat{A}_p) \to \pi_1(\mathrm{Spec}\ A_p)$ is injective.*

Proof. Since $\hat{A}_p/p \cong A_p/p$, Spec $\hat{A}_p \to$ Spec A_p is unramified and faithfully flat. Hence it is an etale covering. □

---

*Associate Professor, General Education and Research Center, Tokyo Polytechnic University,
Received Sept. 10, 2004

Theorem 2.2. *([21]) Let $Y = \text{Spec } A$, A a complete noetherian local normal ring with k a residue field, X a proper Y-scheme, $X_0 = A \otimes_A k$, a_0 a geometric point and a the corresponding point of X. Then the canonical homomorphism $\pi_1(X_0, a_0) \to \pi_1(X, a)$ is an isomorphism.*

Corollary 2.3. *Since $\hat{A}_p$ is a complete noetherian local ring, one has $\pi_1(\text{Spec } A_p/p) \cong \pi_1(\text{Spec } \hat{A}_p)$*

Proof. $\hat{A}_p/p \cong A_p/p$. □

Let k be a field and A a k-algebra of finite type. Let $X = \text{Spec } A$. Assume $A \subset Q(A)$.

Let Γ be the absolute Galois group $\pi_1(Q(A), \overline{Q(A)})$. Let $f \in A$. Then $\pi_1(A[f^{-1}])$ is a quotient Γ/N_f of Γ.

Proposition 2.4. *Let S be an irreducible locally noetherian normal scheme with K the function field. Let Ω be an algebraically closed extension of K. Let a' be a geometric point of* Spec K *and a a geometric point of S. Then the homomorphism*

$$\pi_1(\text{Spec } K, a') \longrightarrow \pi_1(S, a)$$

is surjective. The kernel corresponds to the subextension $\bar{K}/K$ composed by finite extensions in Ω unramified over S.

Proof. The maximal etale covering over S induces an unramified extension of K. Let N be the corresponding normal subgroup. One has an isomorphism $\pi_1(S, a) \cong \Gamma/N$. □

Corollary 2.5. *Let $U \subset X$ be an open immersion. Then the homomorphism $\pi_1(U) \longrightarrow \pi_1(X)$ is surjective.*

Proposition 2.6. *Let $f : Y \to X$ be an etale covering and b, a geometric points which corresponds by f over Y , X. Then the homomorphism $\pi_1(Y, b) \to \pi_1(X, a)$ is injective.*

Definition 2.7. *([21]) A morphism $f : X \to S$ is said to be separable if every fibre is reduced.*

Proposition 2.8. *Let $f : X \to S$ be a proper separable morphism between locally noetherian schemes. Then* Spec $f_*\mathcal{O}_X \to S$ *is an etale covering.*

Theorem 2.9. *([21]) Let $f : X \to Y$ be a proper separable morphism with Y locally noetherian connected scheme. Suppose $f_*\mathcal{O}_X = \mathcal{O}_Y$. Let X' be an etale covering of X. There exists an etale covering Y' such that $X' \cong X \times_Y Y'$, if and only if $\overline{X'_y}$ admits a section over $\overline{X_y}$.*

Corollary 2.10. *([21]) Let $\bar{a}$ be a geometric point of $\overline{X_y}$, a its image in X and b its image in Y. Then the the following sequence is exact:*

$$\pi_1(\overline{X_y}, \bar{a}) \to \pi_1(X, a) \to \pi_1(Y, b) \to 1$$

Corollary 2.11. *([21]) Let k be an algebraically closed field, X and Y connected schemes over k. Suppose that X is proper over k and Y is locally noetherian. Let a be a geometric point of X, b a geometric point of Y with values in the same algebraically closed extension K of k. Let $c = (a, b)$. The homomorphism $\pi_1(X \times_k Y, c) \to \pi_1(X, a) \times \pi_1(Y, b)$ is induced by the homomorphisms between the fundamental groups associated to projections $X \times_k Y \to X$ and $X \times_k Y \to Y$. This homomorphism is an isomorphism.*

Proposition 2.12. *([21]) Let $f : X \to Y$ be a proper separable morphism with Y locally noetherian connected scheme. Suppose $f_*\mathcal{O}_X = \mathcal{O}_Y$. The homomorphism $\pi_1(X) \to \pi_1(\mathrm{Spec}\ f_*\mathcal{O}_X)$ is surjective and $\pi_1(\mathrm{Spec}\ f_*\mathcal{O}_X) \to \pi_1(Y)$ is injective.*

Let $X = \mathrm{Spec}\ A$ be a locally noetherian normal irreducible affine scheme and Γ a profinite group $\pi_1(K)$. Let $\pi_1(A[f^{-1}]) = \Gamma/N_f$, where N_f is a normal closed subgroup in Γ. One has $\pi_1(A_p) = \pi_1(\lim\limits_{\to f \notin p} A[f^{-1}]) = \lim\limits_{\leftarrow f \notin p} \pi_1(A[f^{-1}]) = \lim\limits_{\leftarrow f \notin p} \Gamma/N_f$. So $\pi_1(A_p) = \Gamma/\cap_{f \notin p} N_f$. Let N_p denote $\cap_{f \notin p} N_f$. Next one has

$$\pi_1(A_p/p) \hookrightarrow \Gamma/N_p.$$

Hence there exists a subgroup Γ_p of Γ such that $\pi_1(A_p/p) \cong \Gamma_p/N_p$. Let $\Gamma(p)$ denote Γ_p/N_p. Let $p \subset q$ be prime ideals. Since $A_q \subset A_p$, $\pi_1(A_p) \to \pi_1(A_q)$ is surjective. Hence $N_p \subset N_q$. Characterizing N_p, one defines a set $X = \{N_p\}$. One gives the closure of a point N_p, $V(N_p) = \{N_q | N_p \subset N_q\}$. Let M be a normal closed subgroup of Γ such that $N \subset M$. Let $V(M) = \{N_p | M \subset N_p\}$. One has $\Gamma/M \cong \pi_1(\lim\limits_{\to f \in S} A_f)$. Let a be an ideal of A. For $V(a)$ take $\Delta = \cap\{\Gamma_p | \Gamma_p \text{ associate to } p \in V(a)\}$. One has $\Delta/\Delta \cap N \cong \pi_1(A/a)$.

3 Proper Smooth morphisms

Theorem 3.1. *([21]) Let $f : X \to Y$ be a proper smooth morphism. Suppose $f_*\mathcal{O}_X = \mathcal{O}_Y$ and Y is locally noetherian. Assume that y_0 and y_1 are points of Y such that $y_0 \in \overline{y_1}$ and that $\overline{X_0}$ and $\overline{X_1}$ are geometric fibres. Then the specialization homomorphism $\pi_1(\overline{X_1}) \to \pi_1(\overline{X_0})$ is an isomorphism if $k(y_0)$ is of characteristic 0 (resp. $\pi_1(\overline{X_1})^{(p)} \to \pi_1(\overline{X_0})^{(p)}$ if $k(y_0)$ is of characteristic p).*

Proposition 3.2. *Let $f : X \to S$ be a proper smooth surjective morphism between complex varieties. Suppose $f_*\mathcal{O}_X \cong \mathcal{O}_Y$. Let $g : T \to S$ be a morphism of finite type. Then $\pi_1(X \times_S T) \cong \pi_1(X) \times_{\pi_1(S)} \pi_1(T)$.*

Proof. By base change $f_T : X_T \to T$ is a proper smooth morphism. Hence $\pi_1(X \times_S T) \cong \pi_1(X) \times_{\pi_1(S)} \pi_1(T)$. $\square$

Theorem 3.3. *Let $f : X \to S$ be a proper smooth surjective morphism between complex non singular varieties. Suppose that $f_*\mathcal{O}_X \cong \mathcal{O}_Y$ and that $\pi_1(S, a) = 1$. Then $f : X \to S$ is trivial.*

Proof. One denotes the rational function field of S by $R(S)$. Put $T = \mathrm{Spec}R(S)$. Since $f_T : X_T \to T$ is a proper smooth morphism, $\pi_1(X_T) \cong \pi_1(X) \times \pi_1(T)$. We will show a proof in the following several steps. $\square$

Lemma 3.4. *Let X be a non singular variety and U, V open varieties. Assume* $\mathrm{codim}(U \cup V, X) \geq 2$. *Then the following commutative square is cartesian:*

$$\begin{array}{ccc} \pi_1(U \cap V) & \to & \pi_1(U) \\ \downarrow & & \downarrow \\ \pi_1(V) & \to & \pi_1(X) \end{array}$$

Proof. Let Γ be the absolute Galois group of the function field $R(X)$ of X and N_X, N_U, N_V normal closed subgroups of Γ which correspond to varieties X, U, V, respectively. Since $N_U/N_U \cap N_V \cong N_U N_V/N_V$, one has the map of the following exact sequences the lower square of which is cartesian:

$$\begin{array}{ccc} 1 & & 1 \\ \downarrow & & \downarrow \\ N_U/N_U \cap N_V & \cong & N_U N_V/N_V \\ \downarrow & & \downarrow \\ \Gamma/N_U \cap N_V & \to & \Gamma/N_V \\ \downarrow & & \downarrow \\ \Gamma/N_U & \to & \Gamma/N_U N_V \\ \downarrow & & \downarrow \\ 1 & & 1 \end{array}$$

$\square$

Corollary 3.5. *Let $f : X \to S$ be a surjective separable connected morphism between non singular varieties. Let U be an open subvariety of X. Assume that $f(U) = S$. Let η be the generic point of S and X_η, U_η generic fibres of X/S, U/S, respectively. Then the following commutative square is cartesian:*

$$\begin{array}{ccc} \pi_1(U_\eta) & \to & \pi_1(U) \\ \downarrow & & \downarrow \\ \pi_1(X_\eta) & \to & \pi_1(X) \end{array}$$

Proof. Let Γ be the absolute Galois group of the function field $R(X)$ of X and N_X, N_U, N_{X_η} normal closed subgroups of Γ which correspond to varieties X, U, X_η, respectively. Let W be an open subvariety of S. One has $N_U/N_U \cap N_{X_W} \cong N_U N_{X_W}/N_{X_W}$. Since $\underleftarrow{\lim} W = \eta$, one has $\underleftarrow{\lim}_W N_U/N_U \cap N_{X_W} \cong \underleftarrow{\lim}_W N_U N_{X_W}/N_{X_W}$. Since $N_U/N_U \cap N_{X_\eta} \cong N_U N_{X_\eta}/N_{X_\eta}$, one has the map of the following exact sequences the lower square of which is cartesian:

$$\begin{array}{ccc}
1 & & 1 \\
\downarrow & & \downarrow \\
N_U/N_{U_\eta} & \cong & N_X/N_{X_\eta} \\
\downarrow & & \downarrow \\
\Gamma/N_{U_\eta} & \to & \Gamma/N_{X_\eta} \\
\downarrow & & \downarrow \\
\Gamma/N_U & \to & \Gamma/N_X \\
\downarrow & & \downarrow \\
1 & & 1
\end{array}$$

$\square$

Lemma 3.6. *Let $f : X \to S$ be a proper smooth connected surjective morphism between varieties. Then one obtains the following commutative diagram:*

$$\begin{array}{ccc}
1 & & 1 \\
\downarrow & & \downarrow \\
\pi_1(\overline{U}_\eta) & \cong & \pi_1(\overline{X}_\eta) \\
\downarrow & & \downarrow \\
\pi_1(U_\eta) & \to & \pi_1(X) \\
\downarrow & & \downarrow \\
\pi_1(\eta) & \to & \pi_1(S) \\
\downarrow & & \downarrow \\
1 & & 1
\end{array}$$

Proof. One has the following commutative diagram:

$$\begin{array}{ccc}
1 & & 1 \\
\downarrow & & \downarrow \\
\pi_1(\overline{X}_\eta) & \cong & \pi_1(\overline{X}_\eta) \\
\downarrow & & \downarrow \\
\pi_1(X_\eta) & \rightarrow & \pi_1(X) \\
\downarrow & & \downarrow \\
\pi_1(\eta) & \rightarrow & \pi_1(S) \\
\downarrow & & \downarrow \\
1 & & 1
\end{array}$$

Thus one interprets the commutative diagram above in the following one:

$$\begin{array}{ccc}
1 & & 1 \\
\downarrow & & \downarrow \\
M/N_X & \cong & N/N_{X_\eta} \\
\downarrow & & \downarrow \\
\Gamma/N_X & \rightarrow & \Gamma/N_{X_\eta} \\
\downarrow & & \downarrow \\
\Gamma/M & \rightarrow & \Gamma/N \\
\downarrow & & \downarrow \\
1 & & 1
\end{array}$$

Hence one has $M/N_X \cong N/N_{X_\eta}$.

This implies $M/N_U \cong N/N_{U_\eta}$.

Consider the following commutative diagram:

$$\begin{array}{ccc}
1 & & 1 \\
\downarrow & & \downarrow \\
N_{X_\eta}/N_{U_\eta} & \cong & N_X/N_U \\
\downarrow & & \downarrow \\
N/N_{U_\eta} & \rightarrow & M/N_U \\
\downarrow & & \downarrow \\
N/N_{X_\eta} & \cong & M/N_X \\
\downarrow & & \downarrow \\
1 & & 1
\end{array}$$

Since the lower and upper horizontal arrows are isomorphic, the middle horizontal arrow is an isomorphism by the precedent lemma. $\square$

Lemma 3.7. *Let $g : T \to S$ be an arbitrary morphism and U an arbitrary open subvariety of X. Assume $\pi_1(U_T) = \pi_1(\overline{U}_\eta) \times \pi_1(T)$. Then there exists a variety F such that $X \cong F \times S$ as S-scheme.*

Hence we complete a proof of the theorem above.

We use the tools of L.Breen([8],[9],[10]) in the proof of the following theorem.

Theorem 3.8. *Let $f : X \to S$ be a proper smooth surjective morphism between non singular complex varieties. Assume*

(a) $f_*\mathcal{O}_X = \mathcal{O}_S$

(b) the generic general fibre of $f : X \to S$ is of general type.

(c) $S = S_1 \times S_2$, where S_1, S_2 are varieties.

(d) for any point $b \in S_2$ the restriction $f_{1b} : X_{1b} \to S_1 \times \{b\}$ of $f : X \to S$ has no fixed part over S_1.

Then there exists S_3 such that for any point $a \in S_1$ the base change $f_{2a,S_3} : X_{2a,S_3} \to \{a\} \times S_3$ of the restriction of $f_{2a} : X_{2a} \to S_2 \times \{a\}$ is trivial.

Proof. We will give an outline of a proof in several steps in the following lemmas and propositions. □

Proposition 3.9. *Let $1 \to G \to E \to P \to 1$ be an extension of profinite groups. There exist the following commutative diagram and exact sequence:*

$$\begin{array}{ccccccccc} 1 & \to & G & \to & E & \to & P & \to & 1 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 1 & \to & \mathrm{Inn}G & \to & \mathrm{Aut}G & \to & \mathrm{Out}G & \to & 1 \end{array}$$

and

$$0 \to H^1(P, \mathrm{Z}G) \to H^0(P, (G \to \mathrm{Aut}G)) \to H^0(P, \mathrm{Out}G) \to$$

$$H^2(P, \mathrm{Z}G) \to H^1(P, (G \to \mathrm{Aut}G)) \to H^1(P, \mathrm{Out}G)$$

Lemma 3.10. *Let $1 \to G \to E \to P \to 1$ be an extension of profinite groups. Let $Q \triangleleft P$ be a normal closed group of G. The following sequence is exact:*

$$1 \to H^1(P/Q, \mathrm{Z}G^Q) \to H^1(P, \mathrm{Z}G) \to H^1(Q, \mathrm{Z}G)^{P/Q}$$

Proof. It is the Edge sequence of Serre Spectral sequence([31]). □

In particular, one has the following lemma.

Lemma 3.11. *Let $P = P_1 \times P_2$. The following sequence is exact: for $\{i,j\} = \{1,2\}$*

$$0 \to H^1(P_j, \mathrm{Z}G^{P_i}) \to H^1(P, \mathrm{Z}G) \to H^1(P_i, \mathrm{Z}G)^{P_j}$$

Lemma 3.12. *For $\xi \in H^1(P, (G \to \mathrm{Aut}G))$, let ξ_i be the image element of the following composite homomorphism*

$$H^1(P, (G \to \mathrm{Aut}G)) \overset{res}{\to} H^1(P_i, (G \to \mathrm{Aut}G)) \to H^1(P_i, \mathrm{Out}G)$$

Then there exists a profinite subgroup P_i^* of P_i such that ξ_i is a distinguished element of $H^1(P_i^*, \mathrm{Out}G)$. Hence there exists $\zeta_i \in H^2(P_i^*, \mathrm{Z}G)$ such that the image of ζ_i is res(ξ) by $H^2(P_i^*, \mathrm{Z}G) \to H^1(P_i^*, (G \to \mathrm{Aut}G))$.

Proof. Since Out$G$ is a finite group, it is possible to put $P_i^* = \ker(\xi_i : P_i \to \mathrm{Out}G)$. □

Lemma 3.13. *One can take a profinite subgroup P_i^{**} of P_i^* such that $H^1(P_i^{**}, \mathrm{Z}G) = 0$ for $i = 1.2$. Denoting by $P^{**} = P_1^{**} \times P_2^{**}$. One has $H^1(P^{**}, \mathrm{Z}G) = 0$.*

Proof. Take $P_i^{**} = \bigcap \ker(P_i^* \to \mathrm{Z}G) \supset [P_i^*, P_i^*] \neq 1$. Then $Hom(P^{**}, \mathrm{Z}G) = \Pi_{i \in 1,2} Hom(P_i^{**}, \mathrm{Z}G) = 1$. □

Lemma 3.14. *([31]) Let $Q \triangleleft P$ be a normal profinite subgroup of P. If $H^1(P, \mathrm{Z}G) = 0$, then there exists an exact sequence:*

$$0 \to H^2(P/Q, \mathrm{Z}G^Q) \to H^2(P, \mathrm{Z}G) \to H^2(Q, \mathrm{Z}G)^{P/Q}$$

Proof of theorem. Replace P^{**}, P_i^{**}, P_j^{**} by P, P_i, P_j, respectively. Hence one can assume $H^1(P, \mathrm{Z}G) = 0$, $H^1(P_i, \mathrm{Z}G) = 0$, $H^1(P_j, \mathrm{Z}G) = 0$. Let $\xi \in H^1(P, (G \to \mathrm{Aut}G))$ be an extension :$1 \to G \to E \to P \to 1$ as in the theorem. The image of ξ by $H^1(P, (G \to \mathrm{Aut}G)) \to H^1(P, \mathrm{Out}G)$ is a distinguished element.

Hence there exists an element $\zeta \in \overline{H}^2(P, \mathrm{Z}G)$ such that $\alpha(\zeta) = \xi$ by $\overline{H}^2(P, \mathrm{Z}G) \overset{\alpha}{\to} \overline{H}^1(P, (G \to \mathrm{Aut}G))$. Here the overline of $\overline{H}(P, \mathrm{Z}G)$ indicates the reduced cohomology. Consider the following exact sequence:

$$0 \to \overline{H}^2(P_j, \mathrm{Z}G^{P_i}) \overset{\psi_j}{\to} \overline{H}^2(P, \mathrm{Z}G) \overset{\phi_i}{\to} \overline{H}^2(P_i, \mathrm{Z}G)^{P_j}$$

Take the extension $\xi \in H^1(P, (G \to \mathrm{Aut}G))$ of P such that $\phi_1(\xi)$ has no fixed part restricting to P_1. Suppose the image $\zeta - \psi_1\phi_1(\zeta)$ is not a distinguished element in the cohomology $\overline{H}^2(P, \mathrm{Z}G)$. Then the image $\zeta - \phi(\psi_1(\phi_1(\zeta)))$ is a distinguished element in the cohomology $\overline{H}^2(P_1, \mathrm{Z}G)^{P_2}$. Hence there exists an element η_2 such that $\psi_2(\eta_2) = \zeta - \psi_1\phi_1(\zeta)$. Consider the following exact sequence: $0 \to \overline{H}^2(P, \mathrm{Z}G) \to \overline{H}^1(P, (G \to \mathrm{Aut}G)) \to \overline{H}^1(P, \mathrm{Out}G)$. Thus the extension ξ has a non void fixed part restricting to P_1, which is a contradiction. □

Therefore we complete a proof of the theorem above.

References

[1] M. F. Atiyah and F. Hirzebruch, Vector bundles and homogeneous spaces. In *Differential Geometry, Proc. Sympos. Pure Math*, vol. 3, A.M.S. (1961), 7-38.

[2] A. Beilinson. Letter to C. Soulé, 1982.

[3] A. Beilinson. Higher regulators and values of L-functions. *Current Problems in Mathematics*, **24** (1984), 181-238.

[4] A. Beilinson. Height pairing between algebraic cycles. In *K-Theory, Arithmetic and Geometry*, Moscow 1984-86, Lecture Notes in Math 1289, Springer-Verlag (1987), 1-26.

[5] S. Bloch. Algebraic cycles and higher K-theory. *Adv. in Math* **61** (1986), 267–304.

[6] S. Bloch and S. Lichtenbaum. A spectral sequence for motivic cohomology. *Preprint*

[7] L. Breen Extension of abelian sheaves and Eilenberg-MacLane algebras. *Invent. Math.* **9**,(1969), 15-44.

[8] L.Breen Bitorsours et cohomologie non-abelienne. *The Grothendieck Festschrift I*, P. Cartier et al. edts., *Progress in Mathematics*, **86**, (1990) 401-476.

[9] L.Breen Théorie de Shreier supérieure. *Ann. Scient. Ec. Norm. Sup.*, **25**, (1992) 465-514.

[10] L.Breen On the classification of 2-gerbes and 2-stacks. *Astérisque S. M. F.* 1994.

[11] A. Dold and R. Thom, Quasifaserungen und unendliche symmetrische produckte. *Annals of Math*, **68** (1958), 239–281.

[12] W. Dwyer and E. Friedlander. Algebraic and etale K-theory. *Trans. A. M.S.*, **292** (1985), 247-280.

[13] E. Friedlander. Algebraic cocycles on quasi-projective varieties. *Compositio Math.* (1997).

[14] E. Friedlander, O. Gabber. Cycle spaces and intersection theory. In *Topological Methods in Modern Mathematics*, (1993), 325-370.

[15] E. Friedlander and H.B. Lawson. A theory of algebraic cocycles. *Annals of Math*, **136** (1992), 361–428.

[16] E. Friedlander and H.B. Lawson. Duality relating spaces of algebraic cocycles and cycles. *Topology*, **36** (1997), 533–565.

[17] E. Friedlander and H.B. Lawson. Moving algebraic cycles of bounded degree. *Inventiones Math.*

[18] E. Friedlander and V. Voevodsky. Bivariant cycle homology. In *Cycles, Transfers and Motivic Homology Theories*, Annals of Math Studies, Princeton Univ. Press.

[19] W. Fulton Intersection Theory, Ergebnisee, Springer-Verlag 1984

[20] H. Gillet and C. Soulé. Descent, motives and K-theory. *J. reine angew Math*, **478** (1996), 127–176.

[21] A. Grothendieck. Revêtements etales et groupe fondamental. *SGA1*, Lecture Notes in Mathematics **224**, Springer 1971.

[22] A. Grothendieck, P. Berthelot, and L. Illusie. *SGA 6*, Lecture Notes in Mathematics **225**, Springer 1971.

[23] H.B. Lawson. Algebraic cycles and homotopy theory. *Annals of Math*, **129** (1989), 253–291.

[24] M. Levine. Relative Milnor K-theory. *K-Theory*, **6** (1992), 113-175. Correction: *K-Theory*, **9** (1995), 503-505.

[25] M. Levine. Bloch's higher Chow groups revisited. Astérisque **210** (1994), 235–320.

[26] M. Levine. Motivic cohomology and algebraic cycles: a categorical construction, I. *Preprint* (1993).

[27] S. Lichtenbaum. Values of zeta-functions at non-negative integers. In *Number Theory*, Lecture Notes in Math **1068**, Springer-Verlag (1983), 127–138.

[28] Y. Nisnevich. The completely decomposed topology on schemes and associated descent spectral sequences in algebraic K-theory. In *Algebraic K-theory: connections with geometry and topology* 1989, Kluwer Acad Publ, 241–342.

[29] Yu. P. Nesternko and A. Suslin. Homology of the general linear group over a local ring and Milnor K-theory. Izv AN SSSR.

[30] L.Ribes, P. Zalesski. Profinite Groups. *Ergebnisse der Mathematik und ihrer Grenzgebiete*, **40**. 1991 Springer.

[31] J-P. Serre. Cohomologie Galoisienne. Lecture Notes in Mathematics. , **5** 4th Ed. 1973.

[32] C. Soulé. Opérations en K-théorie algébrique. *Canad. J. Math* **37** (1985), 488–550.

[33] A. Suslin. On the K-theory of algebraically closed fields. *Inventiones Math* **73** (1983), 241-245.

[34] A. Suslin. Higher Chow groups and etale cohomology. In: *Cycles, Transfers and Motivic Homology Theories*, Annals of Math Studies, Princeton Univ. Press.

[35] A. Suslin and V. Voevodsky. Singular homology of abstract algebraic varieties. *Inventiones Math* **123** (1996), 61–94.

[36] A. Suslin and V. Voevodsky. Relative cycles and Chow sheaves. In *Cycles, Transfers and Motivic Homology Theories*, Annals of Math Studies, Princeton Univ. Press.

[37] A. Suslin and V. Voevodsky. Bloch-Kato conjecture and motivic cohomology with finite coefficients. In *Cycles, Transfers and Motivic Homology Theories*, Annals of Math Studies, Princeton Univ. Press.

[38] V. Voevodsky. Homology of Schemes. In *Cycles, Transfers and Motivic Homology Theories*, Annals of Math Studies, Princeton Univ. Press.

[39] V. Voevodsky. Triangulated categories of motives over a field. In *Cycles, Transfers and Motivic Homology Theories*, Annals of Math Studies, Princeton Univ. Press.

[40] V. Voevodsky. The Milnor Conjecture. *Preprint.*